

Objective: To establish new identities using the basic trig identities (quotient, reciprocal, Pythagorean and even-odd identities).

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

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$$\sin x = \frac{1}{\csc x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\cot x = \frac{1}{\tan x}$$

Objective: To establish new identities using the basic trig identities (quotient, reciprocal, Pythagorean and even-odd identities).

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Objective: To establish new identities using the basic trig identities (quotient, reciprocal, Pythagorean and even-odd identities).

$$\sin(-x) = -\sin x \quad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \quad \cot(-x) = -\cot x$$

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Establish the Identity:  
 $\csc\theta \cdot \tan\theta = \sec\theta$

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Establish the Identity:  
 $(\csc\theta - 1)(\csc\theta + 1) = \cot^2\theta$

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Establish the Identity:

$$1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$$

Establish the Identity:

$$\frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta} = \tan \alpha \tan \beta$$



Establish the Identity:

$$2 \tan x \sec x = \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x}$$



$$\begin{aligned}
\text{RHS} &= \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} \\
&= \frac{(1 + \sin x) - (1 - \sin x)}{(1 - \sin x)(1 + \sin x)} && \text{Common denominator} \\
&= \frac{2 \sin x}{1 - \sin^2 x} && \text{Simplify} \\
&= \frac{2 \sin x}{\cos^2 x} && \text{Pythagorean identity} \\
&= 2 \frac{\sin x}{\cos x} \left( \frac{1}{\cos x} \right) && \text{Factor} \\
&= 2 \tan x \sec x = \text{LHS} && \text{Reciprocal identities}
\end{aligned}$$

Establish the Identity:

$$\frac{\cos u}{1 - \sin u} = \sec u + \tan u.$$



**Solution** We start with the left-hand side and multiply numerator and denominator by  $1 + \sin u$ .

$$\begin{aligned}\text{LHS} &= \frac{\cos u}{1 - \sin u} \\&= \frac{\cos u}{1 - \sin u} \cdot \frac{1 + \sin u}{1 + \sin u} && \text{Multiply numerator and denominator by } 1 + \sin u \\&= \frac{\cos u (1 + \sin u)}{1 - \sin^2 u} && \text{Expand denominator}\end{aligned}$$